

# Electron teleportation between quantum dots using virtual dark exciton

M. COMBESCOT, O. BETBEDER-MATIBET, V. VOLIOTIS

*INSP, Université Pierre et Marie Curie-Paris 6, Université Denis Diderot-Paris 7, CNRS, UMR 7588, Campus Boucicaut, 140 rue de Lourmel, 75015 Paris, France*

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**Abstract.** – We here propose a mechanism to teleport electrons between quantum dots through the transformation of a virtual bright exciton into a dark exciton. This mechanism relies on the interactions of two composite bosons: a pair of electrons with opposite spins, trapped in two dots and an electron-hole pair in a free exciton coupled to an unabsorbed pump pulse, which makes it “bright” but virtual. This bright exciton first turns “dark” by dropping its electron and stealing the trapped electron with opposite spin through an exchange Coulomb process with the trapped pair. In a second step, the dark exciton “flies” with its electron to the other dot where it turns bright again, by the inverse process. The “Shiva diagrams” for composite boson many-body effects that we have recently introduced, enlighten this understanding.

Spin degrees of freedom as qubits are under intensive studies due to their potential applications in spintronics and quantum computing. Ultrafast optical control of exciton spin in single quantum dots [1] and its coherent control in two dots [2] through electron-hole exchange, *i. e.*, valence-conduction transition [3], have been recently performed. However, to transfer excitons between dots just amounts to transfer an excitation, each electron staying in its dot. To transfer single electrons, not electron-hole pair, is far more a challenge.

We here propose a physical mechanism to teleport a single electron between two semiconductor quantum dots, through the transformation of a virtual bright exciton coupled to an unabsorbed pump pulse, into a dark exciton.

In a 1986 pioneer experiment, Danièle Hulin and coworkers [4] have shown that all photons, including the unabsorbed ones, act on semiconductors — as proven by a blue shift of the exciton line. This shift, which disappears when the unabsorbed pump is off, provides a physical mechanism for ultrafast optical gates. We explained this exciton optical Stark effect [5,6] by the interactions of two composite bosons: the real exciton produced by the absorbed probe photon and a virtual exciton coupled to the unabsorbed pump pulse.

We here show that similar interactions allow to transfer electrons between quantum dots, the probe exciton being here replaced by a pair of opposite spin electrons trapped in two dots. The transfer is insured by the transformation of a virtual bright exciton, coupled to the unabsorbed pump, into a virtual dark exciton, through exchange interactions with the trapped electrons (see Fig.1). These interactions give rise to a singlet-triplet splitting of

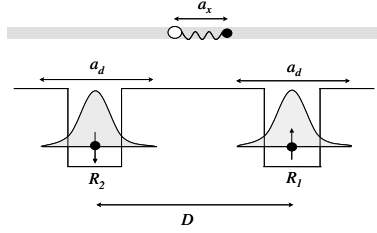


Fig. 1 – Free exciton coupled to unabsorbed photons, in the presence of two electrons (full dots), with opposite spins, trapped in two quantum dots,  $R_1$  and  $R_2$ . Exchanges between the electron of the delocalized exciton and the electrons trapped in the dots make possible the electron transfer

the electron pair. The resulting entanglement of the trapped electron states  $|+1/2, -1/2\rangle$  and  $|-1/2, +1/2\rangle$  allows to teleport a single electron from one dot to the other when the virtual excitons are present, *i. e.*, when the pump is on. So that this transfer can be optically monitored by ultrafast pulses.

For simplicity, we here consider semiconductor with heavy holes only ( $\pm 3/2$  spin) and unabsorbed photons with a  $\sigma_+$  polarization ( $S_z = +1$ ). The virtual free excitons to which these photons are predominantly coupled, thus have a  $(+3/2)$  hole and a  $(-1/2)$  electron, their center-of-mass momentum being the photon one,  $\mathbf{Q}_p \simeq \mathbf{0}$ . After interactions with the trapped electron pair having a  $(+1/2)$  electron on  $\mathbf{R}_1$  and a  $(-1/2)$  electron on  $\mathbf{R}_2$ , the virtual exciton recombines to restore the unabsorbed photon. Electrons being indistinguishable, the  $(-1/2)$  electron which disappears can be the one of the virtual “in” exciton or the one of the dot  $\mathbf{R}_2$ . Let us recall that interesting nonlinearities usually come from processes in which the virtual electron which is created differs from the one which recombines.

To monitor the electron transfer optically, the dot direct coupling through their wave function overlap, must be negligible, so that these dots have to be reasonably far apart. The transfer we propose is based on composite boson interactions. According to our new theory, these interactions generate *two* scatterings [7,8,9]: the energy-like “direct Coulomb scatterings” in which each boson keeps its fermions and the conceptually novel “Pauli scatterings” in which the bosons exchange their fermions without Coulomb process — which makes them dimensionless. All the optical nonlinear effects we have up to now studied [5,10-12] are controlled by Pauli scatterings alone, *i. e.*, processes in which no Coulomb interaction takes place. Consequently, it is not possible to describe these effects correctly through a model Hamiltonian, whatever this model is, since Hamiltonians, by construction, contain energy-like scatterings, while the Pauli scatterings are dimensionless. It turns out that, in the particular case of spatially trapped carriers, these Pauli scatterings have to be mixed with Coulomb processes in order to produce a sizeable transfer, for rather subtle reasons that we now explain.

The set of interactions induced by these two scatterings can be divided in two types:

- (i) In one, the  $(+1/2)$  electron stays in the dots (see Fig.2a). As these processes contain the dot wave function overlap, they produce a negligible transfer when the dots are far apart.
- (ii) In the other, the  $(+1/2)$  electron goes with the hole to form a dark exciton (see Fig.2b). These processes contain the overlaps of the wave function of each dot with the electron wave function in a virtual free exciton. Since the exciton center-of-mass is delocalized over the whole sample, these overlaps differ from zero (see Fig.1), although very small. This is compensated

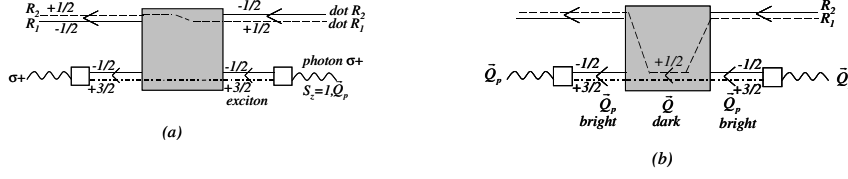


Fig. 2 – In the black box, the composite boson made of the electron-electron pair trapped in the dots, interacts with the composite boson made of an electron-hole pair (exciton) coupled to a  $\sigma_+$  photon, in such a way that the  $(+1/2)$  electron of the dot  $\mathbf{R}_1$  ends on the dot  $\mathbf{R}_2$ . This  $(+1/2)$  electron can either stay in the dots, as in (a), or join the  $(+3/2)$  hole to form a dark exciton, as in (b).

by the large number of such processes, since, along with changing its electron, the exciton also changes its momentum from  $\mathbf{Q}_p$  to  $\mathbf{Q}$ , the excess being provided by the dots.

Our new “Shiva diagrams” for composite boson interactions, here a trapped electron-electron pair and a free exciton, allows to visualize the physics of this electron transfer (see Fig.3). Dimensional arguments allow to grasp the set of microscopic interactions necessary to produce it: We look for a singlet-triplet splitting, *i.e.*, an energy-like quantity. All processes linear in pump intensity contain two energy-like semiconductor-photon couplings (see Fig.2). As the splitting comes from composite boson interactions, it must also contain composite boson scatterings. Pauli scatterings being dimensionless, no energy denominators are needed when they take place. On the opposite, each Coulomb scattering must appear with an energy denominator, which can only be the energy difference between the state 0, made of the trapped pair plus the photon, and one of the various intermediate states appearing in the process at hand. Two relevant ones are the state  $\alpha$  just after the virtual exciton creation and the state  $\beta$  just before its recombination (see Fig.3). They are such that  $\mathcal{E}_0 - \mathcal{E}_\alpha = \mathcal{E}_0 - \mathcal{E}_\beta = -\delta$ , where  $\delta$  is the exciton-photon detuning. A third relevant denominator is  $\mathcal{E}_0 - \mathcal{E}_\gamma$ , where  $\gamma$  is the intermediate state having a dark exciton (see Fig.3c).

Dimensional arguments then show that the process of Fig.3a, which just contains one Pauli scattering, can only have one energy denominator,  $\mathcal{E}_0 - \mathcal{E}_\alpha$ , so that it is the dominant one at large detuning. However, as in it appears the overlap of the  $(\mathbf{R}_1, \mathbf{R}_2)$  dots, its contribution to the singlet-triplet splitting is negligible for far apart dots. Similarly, the process of Fig.3b, which, in addition, contains one Coulomb scattering, has two energy denominators,  $\mathcal{E}_0 - \mathcal{E}_\alpha$  and  $\mathcal{E}_0 - \mathcal{E}_\beta$ , but the dot overlap is still present. To avoid it, the state  $\gamma$  with its dark exciton, has to appear through an energy denominator  $\mathcal{E}_0 - \mathcal{E}_\gamma$ , so that two Coulomb interactions are at least needed to produce a sizeable transfer. In order to also avoid the dot overlap for the  $(-1/2)$  electrons, we end with the virtual “in” exciton leaving its  $(-1/2)$  electron to the dot  $\mathbf{R}_1$  and the virtual “out” exciton having taken its  $(-1/2)$  electron from the dot  $\mathbf{R}_2$  (see Fig.3c).

The “Shiva diagram” of Fig.3c thus corresponds to the large detuning dominant term of the singlet-triplet splitting, linear in pump intensity. It describes the following physics: A  $\sigma_+$  photon with momentum  $\mathbf{Q}_p$  creates a virtual exciton with same momentum, made of  $(+3/2)$  hole and  $(-1/2)$  electron. This exciton has a direct Coulomb scattering with the trapped pair, followed by a Pauli scattering, *i.e.*, a carrier exchange with the dot  $\mathbf{R}_1$ , in order to become dark. In a second step, the dark exciton turns bright again by an electron exchange with the dot  $\mathbf{R}_2$ , followed by a direct Coulomb scattering. In this process, the intermediate dark exciton thus teleports the  $(+1/2)$  electron from dot  $\mathbf{R}_1$  to dot  $\mathbf{R}_2$ .

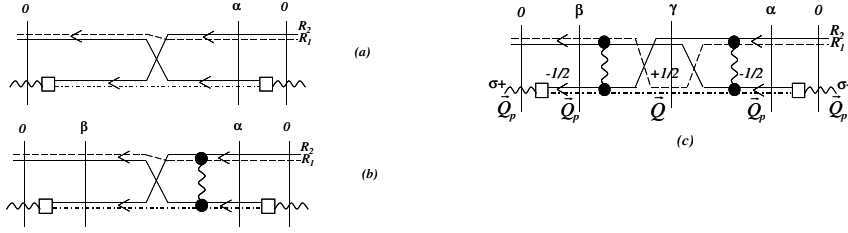


Fig. 3 – “Shiva diagrams” for the set of Pauli and direct Coulomb scatterings leading to transfer the  $(+1/2)$  electron from the dot  $\mathbf{R}_1$  to the dot  $\mathbf{R}_2$ . The processes (a) and (b), having zero and one direct Coulomb scattering, make appearing the dot wave function overlap, so that their contributions to the singlet-triplet splitting is negligible. In (c), which contains two Coulomb scatterings, the intermediate state  $\gamma$  has a dark exciton, while the  $(-1/2)$  electron which regenerates the unabsorbed photon  $\sigma_+$ , differs from the photocreated one, as usual in optical nonlinearities.

The energy denominator associated to this intermediate state  $\gamma$  can stay essentially as small as the detuning  $\delta$  if the trapped electrons and the exciton stay in their ground states. The exciton can however change its momentum from  $\mathbf{Q}_p \simeq \mathbf{0}$  to  $\mathbf{Q}$  without too much cost in energy if the exciton center-of-mass mass,  $M_X$ , is large enough. This leads to  $\mathcal{E}_0 - \mathcal{E}_\gamma = -\delta - \hbar^2 Q^2 / 2M_X$ .

As shown below, the splitting of the electron pair, which corresponds to the “Shiva diagram” of Fig.3c, reads as

$$\Delta \simeq \Omega_0 \frac{1}{-\delta} \sum_{\mathbf{Q}} C_{\mathbf{Q}}(\mathbf{R}_2; \mathbf{R}_1) \frac{1}{-\delta - \hbar^2 Q^2 / 2M_X} C_{\mathbf{Q}}^*(\mathbf{R}_1; \mathbf{R}_2) \frac{1}{-\delta} \Omega_0^* . \quad (1)$$

$\Omega_0$  is the laser coupling to the ground state exciton,  $C_{\mathbf{Q}}(\mathbf{R}_1; \mathbf{R}_2)$  the exchange Coulomb scattering between exciton and trapped electron pair, the exciton exchanging its electron with the dot  $\mathbf{R}_1$ . Its exact value, obtained through the microscopical procedure explained below, is given in eq. (11) in terms of the exciton and dot wave functions, so that it is model dependent.

In the absence of precise experiments, let us concentrate on qualitative behaviors with respect to the physically relevant parameters of the problem, namely the dot distance  $D$ , the photon detuning  $\delta$ , the free electron-hole pair Rabi energy  $\Omega$ , the exciton center-of-mass and relative motion masses,  $M_X$  and  $m_X$ , and the dot and exciton spatial extensions  $a_d$  and  $a_x$  (see Fig.1). For  $Q_p D \simeq 0$ , it is possible to show that  $C_{\mathbf{Q}}(\mathbf{R}_1, \mathbf{R}_2)$  varies with the dot separation as  $e^{i\mathbf{Q} \cdot \mathbf{D}}$ . This leads to a singlet-triplet splitting which behaves as

$$\Delta \simeq \frac{-e^2}{a_x} \frac{\Omega^2}{\delta^2} \frac{M_X}{m_X} \left( \frac{a}{a_x} \right)^d \left( \frac{a}{b_\delta} \right)^{d-2} \left( \frac{b_\delta}{D} \right)^{\frac{d-1}{2}} e^{-D/b_\delta} , \quad (2)$$

where  $d = (2, 3)$  is the space dimension,  $b_\delta$  the “detuning length” defined as  $\delta = \hbar^2 / 2M_X b_\delta^2$  and  $a = \inf(a_d, a_x)$ , equation (2) being valid for rather small detunings ( $b_\delta > a' = \sup(a_d, a_x)$ ). As expected for effects coming from unabsorbed photons, a large splitting requires a small photon detuning  $\delta$  and a large Rabi energy  $\Omega$ , *i. e.*, a powerful pump and a large valence-conduction dipolar matrix element. It is also appropriate to use quantum wells with dots as large as the exciton (to optimize the dot-exciton overlap).

A rough estimate of the transfer time  $T \simeq \hbar/\Delta$  obtained from eq. (2), leads to a few hundreds of picoseconds for reasonable values of the parameters, namely quantum wells having two dots at  $D \simeq 100\text{nm}$ , and  $a_d \simeq a_x \simeq 150\text{\AA}$ , so that  $e^2/a_x \simeq 10\text{meV}$  while  $m_X/M_X \simeq 0.1$ ,  $\Omega \simeq 0.1\text{meV}$  and  $\delta \simeq 0.5\text{meV}$ .

Sham and coworkers [13] have suggested to use unabsorbed photons to exchange spins between dots. However, by lack of appropriate tools to treat interactions with composite excitons properly, their approach can only be phenomenological: They start with a model spin-spin Hamiltonian for trapped and delocalized electrons. This not only misses the Coulomb attraction of the virtual hole responsible for the exciton formation — which essentially kills the electron repulsion — but also the *dimensionless* Pauli scatterings, responsible for all optical nonlinearities. The aim of their second work [14] is to provide a “microscopic” derivation of the spin-spin coupling, through a two-fluid *model* for trapped and delocalized electrons. This is again highly phenomenological, since the “itinerant” (or free) electron states form a complete set for electrons, so that “localized” electrons, which can be written as a sum of “itinerant” electrons, do not differ from them. Consequently, the basis they use is overcomplete. Moreover, the scatterings for interactions between these “two fluids”, on which rely all their results, are not explicitly given in ref. [14]: We do not see how they can be properly derived from a really microscopic approach, *i. e.*, an approach just using the semiconductor Hamiltonian we use.

Let us outline our fully microscopic procedure. It relies on a natural extension of our many-body theory for composite excitons to composite bosons which are not Hamiltonian eigenstates [9]. We start with the bare semiconductor Hamiltonian,  $(H_{\text{sc}} = h_e + h_h + V_{ee} + V_{hh} + V_{eh})$ , and the localization potentials of the two dots,  $(w_{\mathbf{R}_1} + w_{\mathbf{R}_2})$ . As in ref. [12], we introduce the two complete sets of creation operators for one-electron trapped states,  $(h_e + w_{\mathbf{R}_j} - \epsilon_\mu^{(e)}) a_{\mathbf{R}_j\mu s}^\dagger |v\rangle = 0$  with  $j = (1, 2)$  and  $s = (\pm 1/2)$ , with  $|v\rangle$  being the vacuum state.  $\mu$  characterizes the quantum level of the electron in the dot,  $\epsilon_\mu^{(e)}$  being its energy. From them, we construct the composite boson creation operators for trapped electron pairs,  $A_n^\dagger(s_2) = a_{\mathbf{R}_1\mu_1 s_1}^\dagger a_{\mathbf{R}_2\mu_2 s_2}^\dagger$  with  $n = (\mu_1, \mu_2)$ . They are not eigenstates of the system Hamiltonian,  $H'_{\text{sc}} = H_{\text{sc}} + w_{\mathbf{R}_1} + w_{\mathbf{R}_2}$ , since

$$\left[ H'_{\text{sc}} - E_n^{(ee)} \right] A_n^\dagger(s_2) |v\rangle = v_n^\dagger(s_2) |v\rangle \neq 0, \quad (3)$$

where  $E_n^{(ee)} = \epsilon_{\mu_1}^{(e)} + \epsilon_{\mu_2}^{(e)}$ . The RHS of eq. (3) is however small for ground state electrons,  $n = 0 = (\mu_0, \mu_0)$ , if the dots are far apart. The other composite bosons are the free excitons,  $\left[ H_{\text{sc}} - E_i^{(X)} \right] B_{is_i m_i}^\dagger |v\rangle = 0$ , the photons being predominantly coupled to them.

The scatterings between these two composite bosons appear through a set of commutators which are a generalization [9] of those for interacting excitons [7]. The dimensionless “Pauli scatterings”  $\lambda_{n'i'ni}^{(ee-X)}$  are such that

$$\left[ D_{n'n} \begin{pmatrix} s'_2 & s_2 \\ s'_1 & s_1 \end{pmatrix}, B_{is_i m_i}^\dagger \right] = \sum \lambda_{n'i'ni}^{(ee-X)} B_{i's'_i m'_i}^\dagger, \quad (4)$$

where the “deviation-from-boson operator” of the electron pair  $D_{n'n}$  must be defined as

$$\begin{aligned} \left[ A_{n'} \begin{pmatrix} s'_2 \\ s'_1 \end{pmatrix}, A_n^\dagger \begin{pmatrix} s_2 \\ s_1 \end{pmatrix} \right] &= \delta_{n'n} \begin{pmatrix} s'_2 & s_2 \\ s'_1 & s_1 \end{pmatrix} - D_{n'n} \begin{pmatrix} s'_2 & s_2 \\ s'_1 & s_1 \end{pmatrix}, \\ \delta_{n'n} \begin{pmatrix} s'_2 & s_2 \\ s'_1 & s_1 \end{pmatrix} &= \langle v | A_{n'} \begin{pmatrix} s'_2 \\ s'_1 \end{pmatrix} A_n^\dagger \begin{pmatrix} s_2 \\ s_1 \end{pmatrix} | v \rangle, \end{aligned} \quad (5)$$

in order to have  $D_{n'n}|v\rangle = 0$ . The “direct Coulomb scatterings”  $\xi_{n'i'ni}^{(ee-X)}$  appear through

$$\left[ V_n^\dagger \begin{pmatrix} s_2 \\ s_1 \end{pmatrix}, B_{is_i m_i}^\dagger \right] = \sum \xi_{n'i'ni}^{(ee-X)} A_{n'}^\dagger \begin{pmatrix} s_2 \\ s_1 \end{pmatrix} B_{i's'_i m'_i}^\dagger, \quad (6)$$

where the “Coulomb-creation potential” of the electron pair  $V_n^\dagger(s_1^{s_2})$  must be defined as

$$[H'_{sc}, A_n^\dagger(s_1^{s_2})] = E_n^{(ee)} A_n^\dagger(s_1^{s_2}) + v_n^\dagger(s_1^{s_2}) + V_n^\dagger(s_1^{s_2}), \quad (7)$$

with  $v_n^\dagger$  given by eq. (3), in order to have  $V_n^\dagger|v\rangle = 0$ : This insures  $V_n^\dagger$  to describe the interactions of the trapped pair with the rest of the system.

As  $v_0^\dagger|v\rangle \simeq 0$  for far apart dots, the four states  $A_0^\dagger(s_1^{s_2})|v\rangle$  with  $s = (\pm 1/2)$  are essentially degenerate in the absence of pump beam. Their interactions with unabsorbed photons split this fourfold subspace, the changes being obtained from the diagonalization of  $\left[W(\omega_p + E_0^{(ee)} - H'_{sc})^{-1}W\right]$  in this degenerate subspace,  $\omega_p$  being the photon energy and  $W = U + U^\dagger$  the laser-semiconductor coupling. The appropriate way to write  $U$  for  $(\sigma_+, \mathbf{Q}_p)$  photons, is  $U = \sum \Omega_i B_{i,-1/2,3/2}$ , where  $|\Omega_i|$  is the Rabi energy of the exciton  $i = (\nu_i, \mathbf{Q}_p)$ , the ground state one  $|\Omega_o|$  being the largest, by far.

The diagonalization of  $\left[W(\omega_p + E_0^{(ee)} - H'_{sc})^{-1}W\right]$  in the twofold subspace  $A_0^\dagger(s^{-s})|v\rangle$  shows that, for  $Q_p D \simeq 0$ , the  $\sigma_+$  photons split these two states into a triplet and a singlet, according to

$$|\Phi_\pm\rangle = \frac{1}{\sqrt{2}} \left( A_0^\dagger\left(\begin{smallmatrix} -1/2 \\ +1/2 \end{smallmatrix}\right) \pm \frac{E_{-+}}{|E_{-+}|} A_0^\dagger\left(\begin{smallmatrix} +1/2 \\ -1/2 \end{smallmatrix}\right) \right) |v\rangle, \quad (8)$$

their energies being  $\mathcal{E}_\pm = E_0^{(ee)} + E_{++} \pm E_{-+}$ , with

$$E_{s's} = \langle v|U A_0\left(\begin{smallmatrix} -s' \\ s' \end{smallmatrix}\right) \frac{1}{\omega_p + E_0^{(ee)} - H'_{sc}} A_0^\dagger(s^{-s}) U^\dagger|v\rangle. \quad (9)$$

The entanglement of the two states  $A_0^\dagger(s^{-s})|v\rangle$ , with  $s = \pm 1/2$ , resulting from the singlet-triplet splitting  $\Delta = 2E_{-+}$ , gives rise to a transfer time between dots  $T = \pi\hbar/|E_{-+}|$ .

$E_{s's}$  is easy to calculate within our formalism: We first pass the  $A_0^\dagger$  and  $A_0$  over the Hamiltonian, using eq. (7). As for the exciton optical Stark effect [6], this splits  $E_{s's}$  into three terms,  $\alpha_{s's} + \beta_{s's} + \gamma_{s's}$ , which have zero, one and two “Coulomb-creation potentials”. In  $\alpha_{-+}$  and  $\beta_{-+}$  which correspond to the processes of Fig.3a and Fig.3b, appears the overlap of the dot ground states, so that their contributions to  $E_{-+}$  is negligible. The term with two Coulomb creation potentials  $V_0^\dagger$ , which corresponds to the process of Fig.3c, reads

$$\gamma_{-+} = \langle \psi_p|V_0\left(\begin{smallmatrix} +1/2 \\ -1/2 \end{smallmatrix}\right) \frac{1}{\omega_p + E_0^{(ee)} - H'_{sc}} V_0^\dagger\left(\begin{smallmatrix} -1/2 \\ +1/2 \end{smallmatrix}\right) |\psi_p\rangle, \quad (10)$$

where  $|\psi_p\rangle = (\omega_p - H'_{sc})^{-1} U^\dagger|v\rangle$  can be approximated by  $(-\delta)^{-1} \Omega_0^* B_{0,-1/2,3/2}^\dagger|v\rangle$ , with  $0 = (\nu_o, \mathbf{Q}_p)$ , if the photon frequency is far enough from bound exciton resonances. To calculate  $V_0^\dagger(s^{-s}) B_{0,-1/2,3/2}^\dagger|v\rangle$ , we use eq. (6). This generates direct Coulomb scatterings between exciton and trapped electron pair. However, in order to get a sizeable  $\gamma_{-+}$ , it is necessary to avoid the dot wave function overlap for both, the  $(+1/2)$  and the  $(-1/2)$  electrons. So that a Pauli scattering in which the bright “in” exciton and the dot  $\mathbf{R}_1$  exchange their electrons has to be added as well as a Pauli scattering between the dot  $\mathbf{R}_2$  and the black exciton (see Fig.3c). If we then only keep intermediate states  $\gamma$  with the smallest energy, *i. e.*, states in which the exciton and the trapped pair stay in their ground states, we obtain the splitting given in eq. (1), where the Coulomb-exchange scattering precisely reads

$$C_{\mathbf{Q}}^*(\mathbf{R}_1; \mathbf{R}_2) = - \int d\{\mathbf{r}\} \varphi_{\mathbf{R}_2}^*(\mathbf{r}_2) \varphi_{\mathbf{R}_1}^*(\mathbf{r}_e) \phi_{\mathbf{Q}}^*(\mathbf{r}_1, \mathbf{r}_h) v(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_e, \mathbf{r}_h) \\ \times \varphi_{\mathbf{R}_1}(\mathbf{r}_1) \varphi_{\mathbf{R}_2}(\mathbf{r}_2) \phi_{\mathbf{Q}_p}(\mathbf{r}_e, \mathbf{r}_h), \quad (11)$$

where  $v(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_e, \mathbf{r}_h) = v(\mathbf{r}_1 - \mathbf{r}_e) + v(\mathbf{r}_2 - \mathbf{r}_e) - v(\mathbf{r}_1 - \mathbf{r}_h) - v(\mathbf{r}_2 - \mathbf{r}_h)$ , with  $v(\mathbf{r}) = e^2/r$ , is the Coulomb potential between the free exciton and the trapped electrons in their “in” states.  $\varphi_{\mathbf{R}_j}(\mathbf{r})$  is the trapped electron ground state wave function and  $\phi_{\mathbf{Q}}(\mathbf{r}_e, \mathbf{r}_h)$  the exciton ground state wave function,  $\mathbf{Q}$  being the momentum gained by the exciton in this Coulomb exchange process, the increase from  $\mathbf{Q}_p$  to  $\mathbf{Q}$  being provided by the dots. For  $Q_p D \simeq 0$ , in a size  $L$  sample, a bare dimensional analysis leads to  $C_{\mathbf{Q}}^*(\mathbf{R}_1; \mathbf{R}_2) \simeq (e^2/a)(a/L)^d e^{i\mathbf{Q} \cdot \mathbf{D}} f(Qa')$ , where  $f(x)$  is a slowly decreasing function,  $(a, a')$  being defined below eq. (2). The splitting given in eq. (2) then follows from the link between exciton and free pair Rabi energies, namely  $\Omega_o = \Omega(L/a_x)^{d/2}$ , and the fact that  $f(Qa')$  can be replaced by 1 for  $b_\delta < a'$ .

In conclusion, we propose a fully microscopic approach, free from any model Hamiltonian, to the teleportation of electrons between quantum dots through the transformation of a virtual bright exciton coupled to unabsorbed photons, into a virtual dark exciton. This approach relies on a natural extension of our theory for composite exciton many-body effects, to composite bosons which are not semiconductor eigenstates. The “Shiva diagrams” for  $N$ -body exchanges we have recently introduced, make transparent the physics involved.

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